Available online at www.sciencedirect.com
ScienceDirect
Journal of Statistical Planning and Inference 137 (2007) 355–356
journal of statistical planning and inference
www.elsevier.com/locate/jspi

A note on new confidence intervals for the difference between two proportions based on an Edgeworth expansion

Bruno Lecoutre[a,*], Sébastien Faure[a,b]

[a] ERIS, Laboratoire de Mathématiques Raphaël Salem, UMR 6085 C.N.R.S. et Université de Rouen, Avenue de l'Université, BP 12, 76801 Saint-Etienne-du-Rouvray, France
[b] PSICO, Université Charles-de-Gaulle-Lille 3, BP 149, 59653 Villeneuve d'Ascq Cedex, France

Available online 20 February 2006

Abstract

Zhou and Qin [2004. New intervals for the difference between two independent binomial proportions. J. Statist. Plann. Inference 123, 97–115; 2005. A new confidence interval for the difference between two binomial proportions of paired data. J. Statist. Plann. Inference 128, 527–542] "new confidence intervals" for the difference between two treatment proportions exhibit a severe lack of invariance property that is a compelling reason not to use them.

Keywords: Difference between proportions; Confidence intervals; Zhou and Qin's intervals; Invariance; Two-sample design; Paired design

1. A severe lack of invariance

Zhou and Qin (2004, 2005) derived new confidence intervals for the difference between two treatment proportions, respectively, for a two sample design and for a paired design. These intervals are based on an Edgeworth expansion for the studentized difference and they involve a correction of the skewness. Unfortunately, they exhibit a severe lack of invariance property that discourage their use.

Suppose that $[d_l, d_u]$ is the $100(1-\alpha)\%$ confidence interval for the difference $p_1 - p_0$ (given data). Then if we reverse the labeling of the two treatments being compared, the $100(1-\alpha)\%$ confidence interval for $p_0 - p_1$ is not $[-d_u, -d_l]$. In fact, the limits can largely differ from $-d_u$ and $-d_l$, as it can be seen from the two following examples.

For two independent proportions, Zhou and Qin (2004) considered the data counts $(2, 8, 1, 35)$ (SIDS study, p. 108–109). Using their procedure, they computed the two 95% new confidence intervals for $p_1 - p_0$: $[+0.005, +0.516]$ (for their "direct Edgeworth expansion method") and $[-0.024, +0.544]$ (for their "transformation method"). By contrast, the respective intervals for the reverse difference $p_0 - p_1$ are $[-0.373, +0.138]$ and $[-0.401, +0.171]$.

For the paired design, Zhou and Qin (2005) considered the data counts $(4, 6, 3, 3)$ (comparison of the sensitivities of MRI and ultrasound, p. 537). They computed the 90% new confidence interval for $p_1 - p_0$: $[-0.088, +0.502]$. By contrast, the interval for the reverse difference $p_0 - p_1$ is $[-0.424, +0.197]$.

* Corresponding author.
E-mail address: bruno.lecoutre@univ-rouen.fr (B. Lecoutre)
URL: http://www.univ-rouen.fr/LMRS/Persopage/Lecoutre/ (B. Lecoutre).

0378-3758/$ - see front matter
doi:10.1016/j.jspi.2006.01.002

Table 1
Average coverage probabilities of nominal 90% confidence interval for $p_1 - p_0$

n	All cases		$p_1 = p_0$		$p_1 > p_0$		$p_1 < p_0$	
	ZQ	NH	ZQ	NH	ZQ	NH	ZQ	NH
10	0.9114	0.9186	0.9659	0.9543	0.9220	0.9166	0.8947	0.9166
15	0.9031	0.9130	0.9311	0.9219	0.9099	0.9125	0.8933	0.9125
30	0.8990	0.9106	0.9341	0.9375	0.9008	0.9091	0.8933	0.9091
50	0.9002	0.9066	0.9173	0.9273	0.9003	0.9055	0.8983	0.9055
100	0.9003	0.9028	0.9103	0.9109	0.9011	0.9023	0.8984	0.9023

2. A numerical study

Even if arguments could be found to counteract this lack of invariance, its consequence on the coverage probabilities should not be ignored. We replicated the Zhou and Qin numerical studies about the coverage properties of their intervals, with a slight difference. Instead of considering the set of all $(p_0, p_1)=(0.05i, 0.05j)$ for $i, j=1, 2, \ldots, 19$, we generated only parameter values in this set for $i \leqslant j$. Then for $i > j$ we considered the table generated for (j, i) and reversed the labeling of the two treatments. Of course, for an invariant procedure, the coverage probabilities are identical for the two reverse tables of parameter values.

Then we computed separately the coverage probabilities for the three cases $i = j$ $(p_1 = p_0)$, $i < j$ $(p_1 > p_0)$, and $i > j$ $(p_1 < p_0)$. Table 1 displays an extract of our results for a nominal 90% confidence interval in a paired design. For simplicity, we only consider as alternative procedure to the Zhou and Qin (ZQ) interval the Newcombe's hybrid (NH) interval (Newcombe, 1998). Our table is to be compared to Table 3 in Zhou and Qin (2005).

The discrepancy between the coverage probabilities for two reverse tables can be severe. With $n = 15$, we found for instance in our study the coverage probability 0.9321 for the parameter values (p_{11}=0.2868, p_{10}=0.3632, p_{01}=0.0132, p_{00} = 0.3368, hence $p_1 - p_0 = +0.30$), while it is 0.8457 when p_{10} and p_{01} are reversed (hence $p_1 - p_0 = -0.30$). By contrast, the coverage probability of the Newcombe's hybrid interval is 0.9145 in each case. This indicates that the performance of the Zhou and Qin's intervals for small parameter values can be seriously questioned (the discrepancy can be again more severe for more extreme parameter values, even with large n).

3. Conclusion

We know no other example of confidence interval procedure for a difference between proportions that exhibits a lack of invariance. For the odds ratio and relative risk, a well-known example of a procedure that is not invariant under parameter transformation is the Bayesian highest posterior density credible interval. We agree with Agresti and Min (2005) who argued that this is "a fatal disadvantage".

References

Agresti, A., Min, Y., 2005. Frequentist performance of Bayesian confidence intervals for comparing proportions in 2×2 contingency tables. Biometrics 61, 515–523.

Newcombe, R.G., 1998. Improved confidence intervals for the difference between binomial proportions based on paired data. Statist. Med. 17, 2635–2650.

Zhou, X.-H., Qin, G., 2004. New intervals for the difference between two independent binomial proportions. J. Statist. Plann. Inference 123, 97–115.

Zhou, X.-H., Qin, G., 2005. A new confidence interval for the difference between two binomial proportions of paired data. J. Statist. Plann. Inference 128, 527–542.