THE FEASIBILITY OF BAYESIAN METHODS IN ANALYSIS OF VARIANCE:
THE SPECIFIC ANALYSIS APPROACH

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Abstract

Specific Bayesian inferences about linear combinations of means are suggested as routine procedures in analysis of variance. The *specific analysis* approach allows these procedures to be implemented as easily as the traditional t and F tests, even in complex ANOVA designs such as *repeated-measurement* or *cross-over* designs. In particular the non-informative Bayesian solutions are well suited to serve as a concise and objective way of communicating the results. They incorporate the usual frequentist procedures and extend them by direct statements about the importance of effects. Moreover various prior distributions can be investigated to assess the robustness of the conclusions *vis-à-vis* additional information.

Key words

ANOVA; Specific analysis; Bayesian methods; repeated-measurement designs; cross-over designs

1 Introduction

In experimental sciences, the *desirability* of Bayesian methods is more and more recognized. Yet in a field of application as important as analysis of variance, their *feasibility* is still largely questionable for many users. Bayesian procedures have been developed on the subject, but they are generally thought difficult to implement and not included in the commonly available computer packages. In addition, the attitude of Bayesian proponents often looks rigid, as if the use of Bayesian methods entails abandoning the other statistical procedures in use. Furthermore many authors have pointed out the merits of the Bayesian approach in decision making. The consequence is that the contribution of Bayesian inference to experimental data analysis has often been overlooked. This is examplified in clinical research, where analysis of variance is widely applied in complex designs with very specific objectives stated in study

protocols. The analysis of data from early stage protocols is used to direct further developments of the drug, while results of later stage studies with their impact on public health need to be accepted by a large community of scientists and physicians. Although the use of significance testing has been widely debated, it is conventionally accepted as evidence of efficacy. But the experimenter cannot in this way find all the answers to the questions he has posed when devising a complex study, especially in terms of effect size evaluation: see Rouanet, Lecoutre, Bert, Lecoutre, Bernard (1). Obviously, using the Bayesian approach should not result in an abrupt change from the frequentist methods now being employed. Given the widespread use of significance tests, this would be highly unrealistic. As Berry (2) says, "the steamroller of frequentism is not slowed by words.". At the very least, the two methods should co-exist for many years to come. Our attitude is that, rather than replacing current practices, Bayesian procedures should incorporate, extend and refine them. In this perspective, it will be emphasized that routine Bayesian procedures can be used as easily as the familiar t and F tests, and are well suited form commonly used complex experimental designs, such as *repeated-measurement* or *cross-over* designs, thanks to the *specific analysis approach*.

We aim to show in the present paper how this objective can be achieved, with the illustration of two detailed examples where both frequentist and Bayesian procedures are applied. The first trial involves a *repeated-measurement* design to demonstrate that some measurement is proportional to the drug dose. The second trial uses a *cross- over* design with three periods and two sequences to compare two treatments.

For simplicity, we restrict the presentation to the analysis of one- dimensional (one degree of freedom and univariate) effects. But standard Bayesian solutions are also available for multidimensional effects: see Lecoutre (3), Lecoutre and Poitevineau (4), Schervish (5), Rouanet (6).

A Windows interactive computer program, "LeBayesien", which displays and prints Bayesian probability distributions and calculates the corresponding probability statements (together with frequentist significance tests and confidence intervals), can be obtained from the first two authors. It is available on the Internet at the following World Wide Web address:

http://www.univ-rouen.fr/LMRS/Persopage/Lecoutre/pacenglish.htm

The specific analysis approach

Roughly speaking, a specific analysis for a particular effect consists in handling only data that are relevant for it. Most often, the design structure of these relevant data is much simpler that the original design, and the number of "nuisance" parameters involved in the specific inference is drastically reduced. Consequently, in the Bayesian framework, relatively elementary procedures can be applied and realistic prior distributions can be investigated. Furthermore, necessary and minimal assumptions specific to each particular inference are made explicit. When these assumptions are under suspicion, alternative procedures can be easily envisaged: for instance we can do a transformation of the relevant data, or again use solutions that do not assume the equality of variances, etc. Thus, the advantages of the specific analysis approach over the conventional general model approach appear overwhelming for the feasibility of procedures: see Rouanet and Lecoutre (7), Lecoutre (3).

Note that the interest of the specific analysis approach to analysis of variance is often implicitly recognized. In this way, Hand and Taylor (8) suggest systematically deriving relevant data before using commonly available computer packages. In a more particular context Jones and Kenward (9) develop a "*simple and robust analysis for two-group dual designs*" (page 160) which is typically a specific analysis.

2 First example: repeated-measurement design

Eight subjects were successively administered three increasing doses of a drug: 10, 25 and 50 mg. The aim of this study was to demonstrate a proportional relationship between the dose z and the maximal

amplitude of platelet aggregation. The observed means are respectively (the original values have been multiplied by 1000) 30.250 ($z = 10$ mg), 81.125 ($z = 25$ mg), 155.000 ($z = 50$ mg)

2.1 Decomposition of sources of variation

These means can be fitted with the free simple models below, by the least-squares method:
the usual regression line, with equation

$$y = 0.901 + 3.102z$$

the regression line going through the origin

$$y = 3.126z$$

the polynomial of degree 2

$$y = -6.396 + 3.774z - 0.011z^2$$

The parameters of the "general" model are the three means associated with each dose, denoted $\mu_{10\text{mg}}$, $\mu_{25\text{mg}}$, $\mu_{50\text{mg}}$, and the 3×3 matrix of variances and covariances. Here we are interested in the following particular linear combinations of the means:
- the intercept of the usual regression line, designated by ORD,
- the slope of the regression line going through the origin, designated by PRO,
- the coefficient of the term of degree 2 of the polynomial of degree 2, designated by QUA.

These linear combinations are respectively given by the three linear forms:

$$\begin{array}{ll} ORD & [+0.969388\ +0.448980\ -0.418367]' \\ PRO & [+0.003101\ +0.007752\ +0.015504]' \\ QUA & [+0.001667\ -0.002667\ +0.001000]' \end{array}$$

which constitute an orthogonal basis (for the ordinary metric) of the space of linear forms. Consequently we will define the corresponding parameters:

$$\begin{aligned} \delta_{ORD} &= 0.969388\mu_{10\text{mg}} + 0.448980\mu_{25\text{mg}} - 0.418367\mu_{50\text{mg}} \\ \delta_{PRO} &= 0.003101\mu_{10\text{mg}} + 0.007752\mu_{25\text{mg}} + 0.015504\mu_{50\text{mg}} \\ \delta_{QUA} &= 0.001667\mu_{10\text{mg}} - 0.002667\mu_{25\text{mg}} + 0.001000\mu_{50\text{mg}} \end{aligned}$$

To demonstrate an (approximate) proportional relation leads to the statement that the parameters δ_{ORD} and δ_{QUA} both have "negligible" values.

2.2 Specific analyses

The three linear combinations above can be calculated for each of the eight subjects, hence the values given in Table 1. *For each linear combination*, the relevant data set consists of a simple univariate sample, and we can carry out a *specific* inference about the corresponding parent mean. As a specific model we assume the usual *normal* model (with parameters δ and σ) and we simply apply the corresponding inferential solutions. Under this specific model, the derived data set is summarized by the mean d and the standard deviation (corrected from degrees of freedom) s.

subject	10 mg	25 mg	50 mg		ORD	PRO	QUA
1	30	94	12		17.735	2.806	-0.073
2	23	49	80		10.827	1.691	-0.012
3	37	43	151		-8.000	2.789	0.098
4	14	58	85		4.051	1.811	-0.046
5	28	128	171		13.071	3.730	-0.124
6	23	109	175		-1.980	3.629	-0.077
7	28	74	247		-42.969	4.490	0.096
8	59	94	203		14.469	4.059	0.051
mean	30.250	81.125	155.00		0.901	3.126	-0.011
standard deviation	13.371	30.182	56.939		19.771	1.025	0.840

Table 1: First example - basic data and derived data.

Frequentist solutions

The null hypothesis $\delta = 0$ can be tested by means of classical t test. Under this hypothesis, the statistic $t = d/bs$ (with $b^2 = 1/8$ and $b > 0$) is distributed as a Sudent's t with $q = 8 - 1 = 7$ degrees of freedom. The square of t is the F ratio (with 1 and q df) of the respective mean-squares: $ms_1 = (d/ab)^2$, associated with the analysed source of variation, and $ms_2 = (s/a)^2$, associated with the "error term", where a^2 is the sum of squares of the coefficients of the involved linear form over doses.

The traditional analysis of variance table, associated with the considered decomposition of sources of variations, can be reconstituted as a system of specific inferences, each one resting on a separate specific model. This is presented in Table 2.

source	d	s	$t = d/bs$	a	ms_1	ms_2	$F{=}ms_1/ms_2$	p
ORD	+0.901	19.771	+0.129	1.1473	4.93	296.95	0.017	0.901
PRO	+3.126	1.025	+8.627	0.0176	252078.14	3387.36	74.417	0.0001
QUA	-0.011	0.084	-0.368	0.0033	87.56	647.64	0.135	0.724

Table 2: First example - ANOVA table as a system of specific inferences.

Bayesian solutions

The simplest solutions (see for instance Lee (10)) are obtained by assuming a "natural conjugate" prior distribution, that is characterized by the distribution conditional to σ, $\delta|\sigma \sim N(d_0, b_0^2\sigma^2)$ and the marginal distribution for σ $\sigma \sim s_0(\chi^2_{q0}/q_0)^{-1/2}$.

The posterior distribution, given the relevant data, belongs to the same family, with parameters

$$d_1 = \frac{b_0^2 d + b^2 d_0}{b_0^2 + b^2} \qquad b_1^2 = \frac{b_0^2 b^2}{b_0^2 + b^2}$$

$$s_1^2 = \frac{q_0 q_0^2 + q s^2 + (d_0 - d)^2/(b_0^2 + b^2)}{q_0 + q + 1} \qquad q_1 = q_0 + q + 1$$

In particular, the marginal distribution for δ is a *generalized t* distribution $t_{q1}(d_1, b_1^2 s_1^2)$, with center d_1, scale $e_1 = b_1 s_1$, and q_1 df. Thus, in a straightforward way, we can examine the influence of the quantities d_0, b_0, s_0 and q_0 on the (specific) posterior distribution. For instance, for δ_{ORD} (?), let us consider the respective values $d_0 = 0$ and $d_0 = 25$, and $s_0 = 15$ and $s_0 = 25$. With regard

to the investigated conclusion (we want to state that $|\delta_{ORD}|$ is negligible), the values $d_0 = 0$ and $s_0 = 15$ can be considered as favorable or "enthusiastic", and the values $d_0 = +25$ and $s_0 = 25$ as unfavorable or "sceptical". Furthemore we consider for b_0^2 the values 0.01, 1/8 (= b^2), 1 and 100 and for q_0 the values 100, 7 (= q) and 1, that correspond respectively to an increasing uncertainty. For all the combinations of these values, Table 3 gives the limit ϵ such that, for the corresponding posterior distribution $Pr(|\delta_{ORD}| < \epsilon) = 0.95$.

	$b_0 = 0.10$	$b_0 = 0.3536$	$b_0 = 1$	$b_0 = 10$
$d_0 = 0\ s_0 = 15\ q_0 = 100$	$\lvert\delta_{ORD}\rvert < 2.9$	$\lvert\delta_{ORD}\rvert < 7.6$	$\lvert\delta_{ORD}\rvert < 10.2$	$\lvert\delta_{ORD}\rvert < 10.8$
$d_0 = 0\ s_0 = 15\ q_0 = 7$	$\lvert\delta_{ORD}\rvert < 3.5$	$\lvert\delta_{ORD}\rvert < 9.1$	$\lvert\delta_{ORD}\rvert < 12.1$	$\lvert\delta_{ORD}\rvert < 12.9$
$d_0 = 0\ s_0 = 15\ q_0 = 1$	$\lvert\delta_{ORD}\rvert < 4.0$	$\lvert\delta_{ORD}\rvert < 10.3$	$\lvert\delta_{ORD}\rvert < 13.8$	$\lvert\delta_{ORD}\rvert < 14.6$

Table 3: First example - posterior limit ε so that $\Pr(|\delta_{ORD}| < \varepsilon) = 0.95$ as a function of the prior distribution

Standard Bayesian solution

The standard, or "non-informative", solution appears technically as a limiting case, obtained for $b_0 = +\infty$, $s_0 = 0$ and $q_0 = -1$. The posterior distribution depends only on the data: $d_1 = d$, $b_1^2 = b^2$, $s_1^2 = s^2$ and $q_1 = q$. Technically, this solution is particularly easy to compute: the scale $e = bs$ of the posterior distribution of δ is simply the denominator of the t test statistic.

Here we get the posterior standard distribution,

$$\delta \sim t_q(d, b^2 s^2) \sim t_q(d, (d/t)^2) \sim t_7(0.901, (0.901/0.129)^2)$$

hence the statement $Pr(|\delta_{ORD}| < 16.62) = 0.95$.

Let us note that the two "standard Bayesian statements" $Pr(\delta_{ORD} > 0) = 1 - \frac{1}{2}p = 0.549$ and $Pr(-15.63 < \delta_{ORD} < 17.43) = 0.95$
bring the conceptual link with the usual frequentist procedures. On the one hand, the posterior probability for δ to be opposite in sign to the observed value d is equal to the observed *one-sided* $p/2$ level of the t test. On the other hand, the limits of the standard Bayesian credibility interval centered around d coincide with the limits of the frequentist confidence interval.

In the same way, for the parameters δ_{PRO} and δ_{QUA} we get:

$$Pr(\delta_{PRO} > 0) = 0.99997 \text{ and } Pr(\delta_{PRO} > +2.4) = 0.95$$

$$Pr(\delta_{QUA} < 0) = 0.638 \text{ and } Pr(|\delta_{QUA}| < 0.073) = 0.95$$

3 Second example: cross-over design

Let us consider the example reported by Jones and Kenward (9), page 159, about the comparison of two treatments ($t1$ and $t2$) on the blood pressure of hypertensive subjects, using a *cross-over* design with three periods. We will consider here only the data from the two sequence groups $t1$, $t2$, $t2$ ($g1$) and $t2$, $t1$, $t1$ ($g2$). The analysed variable is the systolic blood pressure measured at the end of each period (in mm of mercury). The observed means are given in Table 4.

3.1 Modelling carry-over effects

In order to formalize the mechanisms involved in such a design, a model of the following type on the true means is generally devised: the mean μ_{gpt} (for group g, period p and treatment t) is the

	$p1$	$p2$	$p3$
$g1$ ($f_{g1} = 22$)	$t1$ 157.0909	$t2$ 151.3636	$t2$ 145.9091
$g2$ ($f_{g2} = 27$)	$t2$ 147.1111	$t1$ 150.5556	$t1$ 150.4444

Table 4: Second example - observed means.

sum of the ("fixed") components: the general mean μ, a quantity τ_t depending on the treatment t, a quantity π_p depending on the period p, a quantity $\kappa_{t'}$ depending on the treatment t' administered in the preceding period (consequently null for the first period). This last quantity $\kappa_{t'}$' corresponds to a *residual* or "*carry-over*" effect (here of order 1) of the treament t'. The quantities τ_t and π_p are generally called "treatment effect" and "period effect".

This model can be extended, by assuming for instance that the components τ and/or the components κ depend on the period, or again in the case of three or more period designs that the components κ also depend on the treatments administered in the two (or more) preceding periods (carry-over effects of order 2, 3, etc.). For the sake of simplicity, we shall consider only the simplest model. We then have the situation summarized as follows:

	$p1$	$p2$	$p3$
$g1$	$\mu_{g1p1t1} = \mu + \tau_{t1} + \pi_{p1}$	$\mu_{g1p2t2} = \mu + \tau_{t2} + \pi_{p2} + \kappa_{t1}$	$\mu_{g1p3t2} = \mu + \tau_{t2} + \pi_{p3} + \kappa_{t2}$
$g2$	$\mu_{g2p1t2} = \mu + \tau_{t2} + \pi_{p1}$	$\mu_{g2p2t1} = \mu + \tau_{t1} + \pi_{p2} + \kappa_{t2}$	$\mu_{g2p3t1} = \mu + \tau_{t1} + \pi_{p3} + \kappa_{t1}$

3.2 Decomposition of the sources of variation

In this situation, the comparison of the two treatments ("$t1, t2$"), for the unweighted solution as well as for the weighted (by the group counts) solution, is not orthogonal (for basic metrics associated with the count-measure) to the comparison of the two groups ("$g1, g2$"). This fact leads us to consider for this comparison the following contrast, where [O] denotes the choice of this "orthogonal" solution:

	$p1$	$p2$	$p3$
$g1$	+1/2	-1/4	-1/4
$g2$	-1/2	+1/4	+1/4

$$t1, t2_{[\mathrm{O}]}$$

The interest of the three-period design appears here: the linear combination of the means μ_{gpt} associated with this contrast does not depend on the carry-over effects and is appropriately the difference $\tau_{t1} - \tau_{t2}$.

Moreover it can be noted that this contrast is an interaction contrast between the comparison of the two groups and the comparison "$p1, p2\_p3_{[\mathrm{E}]}$" that compares the first period and the set of the two periods (the symbol "_" indicates the union of modalities). In this case, we consider the unweighted mean for each period, which is denoted by [E]. Hence this leads us to decompose the overall comparison of periods (with two df) as the sum of "$p1, p2\_p3$" and "$p2, p3$". However, in an unbalanced design, the comparison "$p1, p2\_p3_{[\mathrm{E}]}$" is not orthogonal to the comparison "$t1, t2$". If we want an orthogonal decomposition, we must consider the weighted solution (denoted by [N]) for the periods, hence the two partial comparisons "$p1, p2\_p3_{[\mathrm{N}]}$" and "$p2, p3_{[\mathrm{N}]}$". Let us again note that the two interaction contrasts "$g1, g2.p2, p3$" and "$t1, t2.p2, p3$" (defined in canonical way) coincide and are confounded with the comparison "$g1p2\_g2p3, g1p3\_g2p2_{[\mathrm{E}]}$".

Finally we obtain the five following contrasts, which constitute an orthogonal basis of the contrast space on $G \times P$. f_{g1} and f_{g2} denote the group counts.

	$p1$	$p1$	$p3$	
$g1$	+1/3	+1/3	+1/3	+1
$g2$	-1/3	-1/3	-1/3	-1
	1/3	1/3	1/3	

$$g1, g2$$

	$p1$	$p1$	$p3$	
$g1$	+1/2	-1/4	-1/4	+1/2
$g2$	-1/2	+1/4	+1/4	-1/2
	+1	-1/2	-1/2	

$$t1, t2_{[\mathrm{O}]} \equiv g1, g2.p1, p2\_p3_{[\mathrm{E}]}$$

	$p1$	$p1$	$p3$	
$g1$	$+f_{g1}/S$	$-f_{g1}/2S$	$-f_{g1}/2S$	$+f_{g1}/(f_{g1}+f_{g2})$
$g2$	$+f_{g2}/S$	$-f_{g2}/2S$	$-f_{g2}/2S$	$+f_{g2}/(f_{g1}+f_{g2})$
	+1	-1/2	-1/2	

$$p1, p2\_p3_{[\mathrm{N}]}$$

	$p1$	$p1$	$p3$	
$g1$	0	$+f_{g1}/S$	$-f_{g1}/S$	$+f_{g1}/(f_{g1}+f_{g2})$
$g2$	0	$-f_{g2}/S$	$-f_{g2}/S$	$+f_{g2}/(f_{g1}+f_{g2})$
	0	+1	-1	

$$p2, p3_{[\mathrm{N}]}$$

	$p1$	$p1$	$p3$	
$g1$	0	+1/2	-1.2	+1/2
$g2$	0	-1/2	+1/2	-1/2
	0	+1	-1	

$$g1p2\_g2p3, g1p3\_g2p2_{[\mathrm{E}]} \equiv g1, g2.p2, p3_{[\mathrm{N}]} \equiv t1, t2.p2, p3_{[\mathrm{E}]}$$

The three linear forms over periods [1/3 1/3 1/3]', [+1 -1/2 -1/2]' and [0 +1 -1]' define the derivation of relevant data for specific analyses.

3.3 Specific analyses

The specific analyses are a direct extension of the procedures illustrated in the first example. For instance, for the comparison of the two groups, the relevant derived data set consists, for each subject, of the mean of the observations over the three periods, and the situation is simply the comparison of the means of two independent groups. As a general rule, each inference is reduced to the inference about a linear combination $\delta = v_{g1}\mu_{g1} + v_{g2}\mu_{g2}$ of the means of independent groups, where the means μ_g are themselves derived from the means μ_{gpt} defined above (according to the linear form over means considered). As a specific model we assume the usual equivariate normal model (with parameter δ and σ). Under this model, the derived data set for a given contrast is summarized by the linear combination of the observed means $d = v_{g1}x_{g1} + v_{g2}x_{g2}$ and by s^2, within group variance, i.e. the weighted (by df) mean of the two group variances (corrected from degrees of freedom), with $q = f_{g1} + f_{g2} - 2$ df. The constant denoted b^2 depends on the group counts f_{g1} and f_{g2} and on the linear form $[v_{g1}\ v_{g2}]$ over groups, and is defined according to the general formula:

$$b^2 = \sum_{g \in G} \frac{v_g^2}{f_g}$$

Frequentist solutions

The null hypothesis $\delta = 0$ can be tested from the statistic $t = d/bs$ ($b > 0$), which, under this hypothesis, is distributed as a Student's t distribution with q df. The square of this statistic is the ratio:

$$F = t^2 = \frac{m_{s1}}{m_{s2}} = \frac{(d/ab)^2}{(s/a)^2} \text{ [with 1 and } q \text{ df]}$$

where a^2 is the sum of the squares of the coefficients of the linear form over periods considered.

As in the first example, the traditional ANOVA table, given in Table 5, can be reconstituted as a system of specific inferences. By construction, the decomposition is *orthogonal*: the sum of the five mean squares (equal here to the sums of squares) is the sum of squares associated with the overall comparison (with 5 df) of the six means (1740.50). These results are those given by Jones and Kenward in Table 4.11 (page 159), with the following correspondence (the effect of the comparison "$g1, g2$" of the two sequences is included in their source of variation "*Between subjects*"):

Periods	$p1, p2\_p3_{[\mathrm{N}]}$
Direct treatments	$t1, t2_{[\mathrm{O}]}$
Carry-over	$g1p2\_g2p3, g1p3\_g2p2_{[\mathrm{E}]}$

source of variation	d	s	b	$t = d/bs$	a	ms_1	ms_2	F	p
$g1, g2$	+2.08	14.71	0.2872	+0.493	0.5774	157.97	648.74	0.24	0.624
$t1, t2_{[\mathrm{O}]}$	+5.92	16.36	0.1436	+2.520	1.2247	1133.59	178.49	6.35	0.015
$p1, p2\_p3_{[\mathrm{N}]}$	+1.93	16.36	0.1429	+0.825	1.2247	121.50	178.49	0.68	0.414
$p2, p3_{[\mathrm{N}]}$	+2.51	18.22	0.1429	+0.964	1.4142	154.38	166.02	0.93	0.340
$g1p2, g2p3, g1p3\_g2p2_{[\mathrm{E}]}$	+2.67	18.22	0.1436	+1.021	1.4142	173.06	166.02	1.04	0.312

Table 5: Second example - ANOVA table as a system of specific inferences.

The specific analysis of the two contrasts $t1, t2_{[\mathrm{O}]}$ and $g1p2\_g2p3, g1p3\_g2p2_{[\mathrm{E}]}$ corresponds with the procedure exposed by Jones and Kenward as "*a simple and robust analysis for two-group dual designs*" (page 160). Note simply that their contrasts have opposite signs and coefficients divided by two. But the authors consider the t test only, and for instance conclude that: "*clearly, there is no evidence ($p = 0.32$) of a difference in carry-over effects*". Actually, this is a negative result and does not really bring out any evidence in the data that might be positively in favor of a null, or at least negligible, difference in carry-over effects.

Standard Bayesian solution

The specific Bayesian solutions for (δ, σ) are obtained as in the first example. In particular the standard (non-informative) solution is summarized in Table 6. The limit of 7.07 obtained for the absolute value of the difference in carry-over effects is greater than the difference observed between the two treatments (5.92) and can hardly be considered as relatively negligible! Clearly, in order to demonstrate that the difference in carry-over effects is negligible, strong additional information is necessary. Let us consider the "enthusiastic" prior value $d_0 = 0$. Still assuming a non-informative prior for σ, we can compute the limit ϵ so that $Pr(|\delta| < \epsilon) = 0.95$ as a function of b_0. We get for instance, for $b_0 = b = 0.1436$, $\epsilon = 4.45$, and, for $b_0 = b = 0.05$, $\epsilon = 1.82$.

$g1, g2$	$Pr(\delta > 0 = 0.688$	$Pr(\|\delta\| < 9.38) = 0.95$
$t1, t2_{[O]}$	$Pr(\delta > 0 = 0.992$	$Pr(\delta > 1.98) = 0.95$
$p1, p2\_p3_{[N]}$	$Pr(\delta > 0 = 0.793$	$Pr(\|\delta\| < 5.87) = 0.95$
$p2, p3_{[N]}$	$Pr(\delta > 0 = 0.830$	$Pr(\|\delta\| < 6.89) = 0.95$
$g1p2, g2p3, g1p3\_g2p2_{[E]}$	$Pr(\delta > 0 = 0.844$	$Pr(\|\delta\| < 7.07) = 0.95$

Table 6: Second example - standard Bayesian statements.

4 Conclusion

In conclusion, we suggest using specific Bayesian inferences concerning linear combinations of means, carefully selected according to the experimental aims, as routine procedures in analysis of variance. In this perspective, non-informative solutions have a privileged status, since they do not result in an abrupt change from current practice. They provide posterior probabilities as references for "public use", which can serve as a concise and objective ways of communicating the results. Moreover, they incorporate the traditional t and F tests, and extend them by direct statements concerning the importance of effects.

At a later stage, these routine procedures can be extended by analyses involving different prior distributions. Various prior distributions, such as *clinical* (i.e. expressing results from previous studies, or subjective opinion of well-informed individuals), *sceptical* or *enthusiastic* priors, can be investigated, in line with the Bayesian methodology for clinical trials developed by Spiegelhalter, Freedman, and Parmar (11). Thus the robustness of the conclusions *vis-à-vis* additional information can be assessed.

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